

## DOCUMENT RESUME

ED 138 107

FL 008 527

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TITLE On Simple Semantic Spaces and Semantic Categories.  
Working Papers of the Language Behavior Research  
Laboratory, No. 2.  
INSTITUTION California Univ., Berkeley. Language and Behavior  
Research Lab.  
SPONS AGENCY California Univ., Berkeley. Inst. of International  
Studies.; National Inst. of Mental Health (DHEW),  
Rockville, Md.  
PUB DATE Nov 67  
GRANT MH-25703  
NOTE 21p.  
EDRS PRICE MF-\$0.83 HC-\$1.67 Plus Postage.  
DESCRIPTORS \*Componential Analysis; Descriptive Linguistics;  
Distinctive Features; Language Research; Linguistic  
Theory; Psychology; \*Scientific Methodology;  
\*Semantics; \*Vocabulary  
IDENTIFIERS \*Semantic Categories

## ABSTRACT

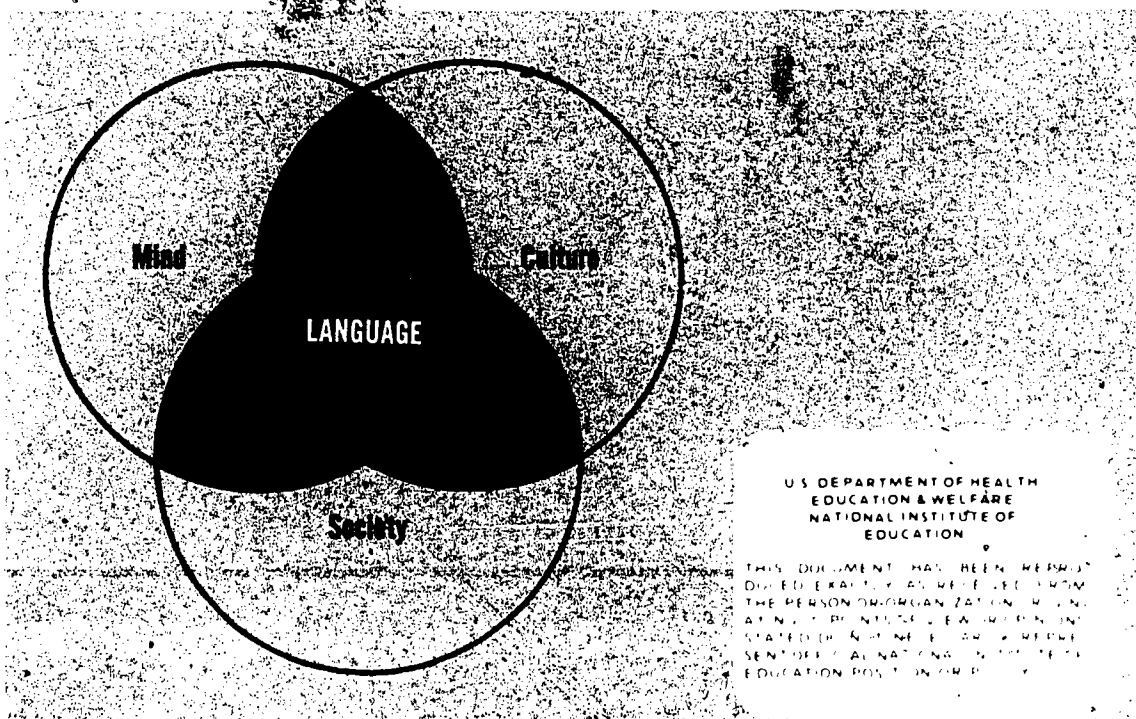
This paper attempts to develop some relatively well-defined notions that may be of use in descriptive semantics and in some areas of cognitive psychology. These notions are intended as explications of certain terms widely used in these fields such as semantic dimension, semantic feature, semantic space, category, conjunctive category, and sememe. The mode of explication is to develop definitions for these terms in an argument similar to a formal axiomatic system, with emphasis on the applicability of terms to empirical data. (CLK)

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ED138107

FL008527

Working Papers of the  
Language Behavior Research Laboratory  
University of California, Berkeley



The Language Behavior Research Laboratory is an interdisciplinary research project at the University of California at Berkeley. It is supported by Research Grant No. MH 25703 from the National Institute of Mental Health. This support is gratefully acknowledged.

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University of California  
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ON SIMPLE SEMANTIC SPACES AND  
SEMANTIC CATEGORIES

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Working Paper No. 2  
Language-Behavior Research Laboratory  
November, 1967

The Language-Behavior Research Laboratory  
is funded by a grant from the Institute  
of International Studies, University of  
California, Berkeley, whose support is  
gratefully acknowledged.

This paper is an attempt to develop some relatively well-defined notions that may be of use in descriptive semantics and in some areas of cognitive psychology. These notions are intended as explications of certain terms now widely used in these fields, for example: semantic dimension, semantic feature, semantic space, category, conjunctive category, sememe, etc. The method of explication is to develop definitions for these terms in an argument that is somewhat akin to a formal axiomatic system. The emphasis, however, is always on the applicability of terms to empirical data, and so mathematical elegance is ruthlessly sacrificed to empirical considerations.

Imagine a meta-language ( $S_1$ ) that contains only "predicators" (names for kinds of things) and the common logical connectives  $\wedge$  ("and"),  $\vee$  ("or"),  $\sim$  ("not"), etc. Each predicator has an extension, the set of objects it names, and an intension, the property or attribute shared by the members of this set.<sup>1</sup>

$S_1$  is a meta-language for talking about some natural language (particularly its sememic stratum) or natural non-linguistic cognitive system. In stratificational linguistics, predicators may be interpreted as emic units at the semantic, not the sememic, level (Lamb, 000, 000). We do not assume the existence of a universal version of  $S_1$  with a set of predicators sufficient for modeling all natural systems of meaning. We consider that the particular predicators in  $S_1$  will vary with the particular natural system of meaning being modeled.

We make two assumptions about  $S_1$ . The first concerns the number of predicates. The second concerns relations among the predicates.

- (1) assumption                      The set of predicates in  $S_1$  is, at most,  
denumerably infinite.

Roughly, this means that we can put the predicates into one-to-one correspondence with the set of positive whole numbers or with some proper subset thereof. The important thing is that we can enumerate the predicates; even if the number of predicates should be infinite, we can still speak of 'the first predicate', 'the second predicate', ..., 'the  $(10^7)$ th predicate', and so on.

Secondly, we assume that, for any particular natural system of meaning under study, there exists a cognitively real partition<sup>2</sup>  $B$  on the set of predicates that has the following two properties: First, if ' $P_i$ ' and ' $P_j$ ' are distinct predicates in the same cell of the partition, then any statement asserting the existence of an object having both the properties  $P_i$  and  $P_j$  is logically false.<sup>3</sup> Secondly, no cell of the partition is empty; that is, every cell contains at least one predicate. This assumption may be stated symbolically.

(2) assumption

There exists a partition  $B = \{c_1, c_2, \dots, c_n\}$  on the set of predicates  $\{P_1, P_2, \dots\}$  such that.

(2.1) For any two distinct predicates  $P_1, P_2$  in an arbitrary cell  $c_k$ , the statement  $(\exists x) P_1(x) \& P_2(x)$  is L-false.

(2.2) For any cell  $c_1$ ,  $\exists (x) x \in c_1 \& x = P_1$  where  $P_1$  is a predicate in  $S_1$ .

We may interpret (2.1) as saying that statements that assign the intensions of two distinct predicates from the same cell to a given object are logically contradictory. From (2.1) it follows that the extensions of two predicates in the same cell are mutually exclusive. In symbols,

(3) For any  $c_k \in B$  and  $P_1, P_j \in c_k$ :

$$\sim (\exists x) P_1(x) \wedge P_j(x).$$

The second part of the assumption (2.2) just rules out empty cells.

The interpretation we have in mind for the cells of the partition  $B$  is that each cell contains predicates corresponding to properties that cannot, in principle, be assigned to the same object. Examples are the various masses in grams an object may have, the two sexes, the number of generations by which two-lineal relations may be separated, the two values of deciduousness, etc.

Since we have assumed, in (1), that the set of all predicates is, at most, denumerably infinite, it follows that the set of predicates constituting any cell  $c_i$  of the partition  $B$  is also at most denumerably infinite. Hence, for each cell  $c_i$  we may enumerate its constituent predicates. (The  $c_i$ 's may, of course, also be enumerated since there is a finite number  $n$  of them.)

We shall use the expression ' $d_j^{(i)}$ ' to refer to the  $j^{\text{th}}$  predicate in the  $i^{\text{th}}$  cell of the partition  $B$ . For example, the third predicate in the fourth cell is represented ' $d_3^{(4)}$ '; the 223<sup>rd</sup> predicate in the  $n^{\text{th}}$  cell is represented ' $d_{223}^{(n)}$ '; the  $x^{\text{th}}$  predicate in the  $y^{\text{th}}$  cell is represented ' $d_x^{(y)}$ '. We refer to the set that is the extension of the predicate ' $d_j^{(i)}$ ' by the expression  $d_j^{(i)}$ .

As may be surmised from the examples in the second paragraph preceeding, we do not wish to limit interpretation to cases in which all objects are necessarily describable by a predicate from every cell. For example, for many objects both predicates in the cell labeled "sex" are irrelevant, for other objects "mass," "generation," etc. are irrelevant. Accordingly, we wish, for each cell  $c_i$  to define the set of objects that possess none of the properties referred to by the predicates in that cell.

For each cell  $c_i$ , we define the set

$$(4) \text{ definition } d_0^{(i)} = \sim (d_1^{(i)} \vee d_2^{(i)} \vee \dots).$$

The set  $d_0^{(i)}$  is the set of things having none of the properties  $d_1^{(i)}, d_2^{(i)}, \dots$ .



We now define a semantic dimension  $D^{(i)}$  as the set whose members are the sets of objects corresponding to the predicators of the cell  $c_i$  plus the set of objects  $d_0^{(i)}$ .

(5) definition  $D^{(i)} = \{d_0^{(i)}, d_1^{(i)}, d_2^{(i)}, \dots\}$ .

Each member  $d_j^{(i)}$  of a semantic dimension  $D^{(i)}$  is a set of objects. All the members  $d_j^{(i)}$  of  $D^{(i)}$  except the member  $d_0^{(i)}$  also correspond to some property of objects, namely of the property that is the intension of the predicator ' $d_j^{(i)}$ '. The expression  $d_0^{(i)}$  may be interpreted as representing the set of objects having the semantic feature of possessing none of the properties referred to by the predicators ' $d_1^{(i)}$ ', ' $d_2^{(i)}$ ', and so on. Accordingly, we shall give all expressions of the form  $d_j^{(i)}$  a second interpretation as semantic features. Each semantic dimension  $D^{(i)}$  contains a zero feature  $d_0^{(i)}$ , which is the feature of "possessing none of the properties corresponding to the predicators ' $d_1^{(i)}$ ', ' $d_2^{(i)}$ ', ...". For example, if  $D^{(1)}$  is the dimension "sex", then  $d_0^{(1)}$  is the semantic feature "sexlessness"; if  $D^{(2)}$  is the dimension "mass", then  $d_0^{(2)}$  is the semantic feature "having no mass," and so on. In the numbered statements we always treat expressions of the form  $d_j^{(i)}$  as sets (of objects), but we shall frequently refer to them in the text as "semantic features."

Each semantic feature refers to a set of objects, and each semantic dimension constitutes a partition of the set of all objects.<sup>4</sup> However, we would like our semantic meta-language to contain a kind of unit corresponding to a set of objects that is minimal in the sense that we cannot further subdivide it, that is, describe any proper subset of it. The basic desideratum for the notion of "semantic point" is that by assigning an object to a unique semantic point we make the finest description of that object that is possible in  $S_1$ . Also, we would like the set of semantic points to constitute a partition of the set of objects. That is, we would like each object to belong to one semantic point and no object to belong to more than one. It is, however, neither necessary nor desirable that there exist an object corresponding to each semantic point.

Accordingly, given a finite number  $n$  of semantic dimensions, we define a semantic point  $p_j$  as the intersection of any  $n$  features, such that exactly one feature is a member of each dimension. In symbols, we define an arbitrary semantic point

(6) Definition

$$p_j = d_{\alpha}^{(1)} \wedge d_{\beta}^{(2)} \wedge \dots \wedge d_{\gamma}^{(n)}$$

$$(d_{\alpha}^{(1)} \in D^{(1)}, d_{\beta}^{(2)} \in D^{(2)}, \dots, d_{\gamma}^{(n)} \in D^{(n)}).$$

By this definition, a semantic point describes a set of objects all of which contain exactly one feature  $d_j^{(i)}$ , possibly the null feature  $d_0^{(i)}$ , from each dimension  $D^{(i)}$ . No object belongs to more than one semantic point, and every object belongs to one. No finer partition of the set of all objects can be derived from the original language  $(S_1)$  than the set of semantic points.

For future convenience, we define one more item of notation. Let the expression  $d_j^{(i)}$  refer to the feature on the  $i^{\text{th}}$  dimension that occurs in the semantic point  $p_j$ . In general, given a semantic point  $p_j$

$$(7) \quad d_j^{(i)} \neq d_j^{(i)},$$

since the point  $p_j$  may contain any feature from dimension  $D^{(i)}$ , not necessarily the  $j^{\text{th}}$  feature in the enumeration  $d_0^{(i)}, d_1^{(i)}, d_2^{(i)}, \dots$ . Using this notation, the defining equation for a semantic point  $p_j$  can be rewritten

$$(8) \quad p_j = \bigcap_{i=1}^n p_j^{(i)}.$$

Recall that  $n$  is the number of cells in the partition  $B$  (=the number of semantic dimensions).

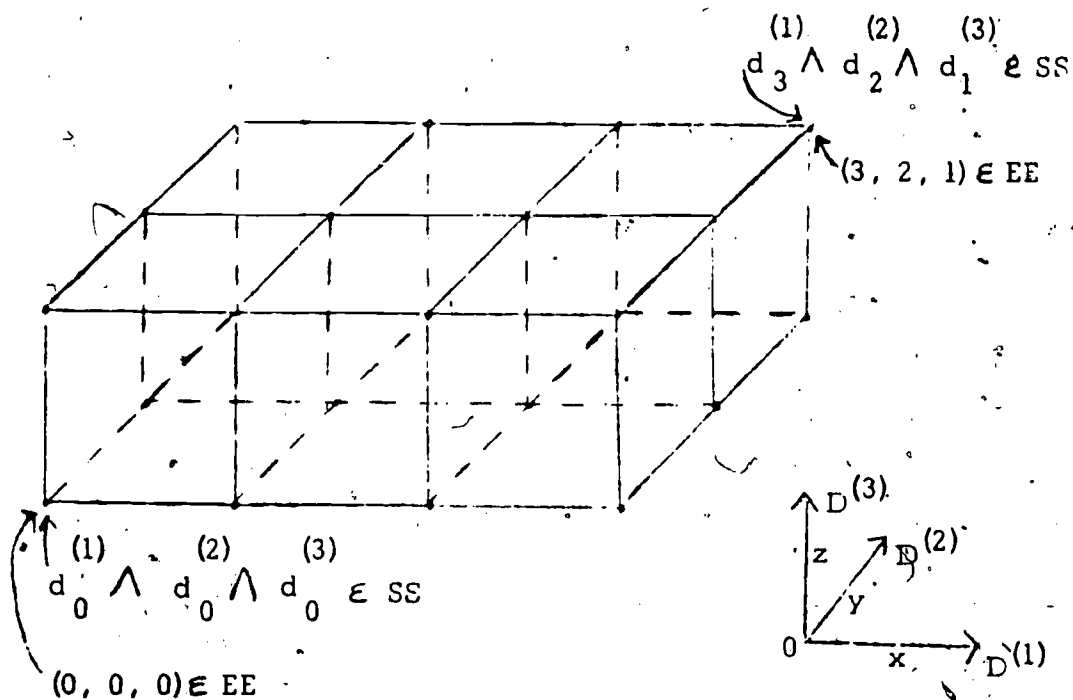
We call the set  $SS$  of semantic points an  $n$ -dimensional semantic space because of an analogy with a subset of the points in Euclidean  $n$ -space.

Consider an arbitrary semantic point  $p_j = d_\alpha^{(1)} \wedge d_\beta^{(2)} \wedge \dots \wedge d_5^{(n)}$ . To any such semantic point there corresponds a unique point  $q_j$  in Euclidean

$n$ -space, namely the point  $(\alpha, \beta, \dots, 5)$ . Any such point  $q_j$  will necessarily have non-negative integers for all its coordinates. Let us call  $EE$  the set of Euclidean points corresponding in the way just mentioned to the points in a semantic space  $SS$ .  $EE$  has the additional property that if it contains the point  $(\alpha, \beta, \dots, 5)$ , then it contains every point  $(a, b, \dots, n)$  where  $a \leq \alpha, b \leq \beta, \dots, n \leq 5$ . For example, if  $SS$  is three dimensional with say 4, 3, and 2 features on dimensions  $D^{(1)}, D^{(2)}$ , and  $D^{(3)}$  respectively, the corresponding set of Euclidean points  $EE$  looks like Figure 1.

Figure 1.

Subsets of points  $EE$  of Euclidean three-space corresponding to a three-dimensional semantic space  $SS$  with one two-feature, one three-feature, and one four-feature dimension.



To each point  $(a, b, c)$  in the sub-space  $EE$  diagrammed in Figure 1., there corresponds the unique semantic point in  $SS$   $d_a^{(1)} \wedge d_b^{(2)} \wedge d_c^{(3)}$ , and conversely. For example to the point  $(3, 2, 1)$  in  $EE$  there corresponds the semantic point  $d_3^{(1)} \wedge d_2^{(2)} \wedge d_1^{(3)}$ .

We noted above that a semantic point gives the finest possible description (in  $S_1$ ) of an object. That is, a semantic point defines a set of objects that cannot be subdivided. We would like to define a semantic category in such a way that it can be interpreted as "any description of a set of objects that is possible in  $S_1$ ." This is easily accomplished by defining a semantic category as the union of any number of semantic points in  $SS$ . Hence for any arbitrary collection of semantic points  $p_1, p_2, \dots, p_m$  in  $SS$  we define the semantic category,

$$(9) \text{ definition } C = p_1 \cup p_2 \cup \dots \cup p_m \Rightarrow \bigcup_{j=1}^m p_j = \bigcup_{j=1}^m \bigcap_{i=1}^n p_j^{(i)}$$

Each semantic point is, of course, itself a category.<sup>5</sup>

The notion conjunctive category has been used and partially defined in several different ways (e.g., Lounsbury, 1964; Bruner et al., 1957).

The essential idea behind all these usages is that conjunctive categories are those that do not involve dependencies among semantic dimensions.

If a category  $C$  is conjunctive, then if  $C$  describes objects with the feature  $d_j^{(i)}$  and objects with the feature  $d_r^{(k)}$  ( $i \neq k$ ), then  $C$  describes at least one object with both features  $d_j^{(i)}$  and  $d_r^{(k)}$ . This amounts to

saying that for conjunctive categories, the union of the intersections (of features) is equal to the intersection of the unions. In symbols, a category  $C$  is conjunctive if and only if

$$(11) \quad C = \bigcup_{j=1}^m \bigcap_{i=1}^n d_{(j)}^{(i)} = \bigcap_{i=1}^n \bigcup_{j=1}^m d_{(j)}^{(i)}$$

Conjunctive categories, so defined, correspond closely to the usual notion. In a conjunctive category there is no "interaction" between the features of different dimension; the dimensions are, in effect, independent. If we construct a new semantic point  $C'$  by intersecting features taken from points that are subsets of a conjunctive category  $C$ , the point  $C'$  is always found to be a subset of  $C$ .

#### EXAMPLE

Assuming  $ES$  contains just two dimensions  $D^{(1)}$ ,  $D^{(2)}$ , the following category is conjunctive:

$$C_1 = (d_3^{(1)} \cap d_2^{(2)}) \cup (d_6^{(1)} \cap d_3^{(2)}) \cup (d_6^{(1)} \cap d_2^{(2)}) \cup (d_3^{(1)} \cap d_3^{(2)}),$$

$$\text{since } C_1 = (d_3^{(1)} \cup d_6^{(1)}) \cap (d_3^{(2)} \cup d_2^{(2)})$$

In the notation frequently used in componential analysis, rewrite  $D^{(1)}$  as  $A$ ,  $D^{(2)}$  as  $B$ ,  $d_j^{(1)}$  as  $a_j$ , and  $\cap$  as concatenation. For example, " $d_3^{(1)} \cap d_2^{(2)}$ " becomes " $a_3 b_2$ ". The above example is now

$$C_1 = Df \ a_3 b_2 \cup a_6 b_3 \cup a_6 b_2 \cup a_3 b_3 = a_{3,6} b_{2,3}, \text{ where}$$

$$a_{3,6} \text{ stands for } a_3 \cup a_6.$$

The semantic points that can be constructed by intersecting features in  $C_1$  are just  $a_{32}b$ ,  $a_{33}b$ ,  $a_{62}b$ , and  $a_{63}b$ , each of which is a subset of  $C_1$ .

Of categories that are not conjunctive, particular attention has been paid to a type called relational (see, for example, Bruner, et al. 1967). When we have explained what we mean by relational category, a threefold classification of semantic categories will have been established: conjunctive, relational and disjunctive. Disjunctive categories are simply those that are neither conjunctive nor relational.

The basic idea of relational categories is that they approximate the defining feature of conjunctive categories -- independence or lack of interaction between semantic dimensions -- with one exception. The exception is that for exactly two dimensions there is interaction between the features, and this interaction is of an orderly kind, i.e., predictable from a simple rule. The presence of this rule, expressing a relation between the features on two semantic dimensions, is the justification for calling such categories "relational."

Accordingly we define relational categories as follows:

A category  $C = \bigcup_{j=1}^m \bigcap_{i=1}^n d_{(j)}^{(i)}$  is relational if and only if

(12) definition

(1) for two distinct semantic dimensions  $D^{(k)}, D^{(r)}$ ,

$$\bigcup_{j=1}^m \bigcap_{i=1}^n d_{(j)}^{(i)} = \bigcap_{i=1}^n \bigcup_{j=1}^m d_{(j)}^{(i)} \quad (i \neq k, i \neq r);$$

(2) for any point  $p_j \in C$ , at least one of the following

holds

(a)  $d_{(j)}^{(k)} \neq d_{(j)}^{(r)}$

(b)  $d_{(j)}^{(k)} \leq d_{(j)}^{(r)}$

(c)  $d_{(j)}^{(k)} \geq d_{(j)}^{(r)}$

Part (1) of definition (12) says in effect that a relational category has the conjunctive property except for two dimensions  $D^{(k)}$  and  $D^{(r)}$ .

Part (2) says that, for each point  $p_j$  included in  $C$ , a stateable relation

(a), (b), or (c) holds between the features on the kth and rth dimensions.



General Discussion and Notes on  
Possible Future Developments.

A componential analysis can be thought of as mapping of a set of lexical units onto or into a semantic space. Various notions in the componential analysis literature can be accommodated. E.g., a perfect paradigm (Kay 1966 and others) can probably be defined in terms of a one-one mapping of a set of lexical items onto the set of points in SS. Wallace and Atkins (1960) notion of an "orthogonal semantic space" is, I think, a one-one mapping of the set of lexical units onto a partition of SS, each cell of which is a conjunctive category.

The key novelty in the present formulation, other than the pretensions to a bit more rigor than usual, is the inclusion of the zero feature  $d_0^{(i)}$  on each semantic dimension. I think this will help in many otherwise difficult empirical applications.

The notion of semantic features as two-faced wigits, with an intension (property) and an extension (class), is a direct adaption from Carnap. Unlike Carnap, who develops a separate calculus for intensions [I think it is fair to say], the present approach sticks with extensions in the formal discussion, but interprets this discussion informally as dealing with properties. This is inelegant, but seems more useful at the current stage of development.

I hope these notes will clear up one persistent confusion in the componential analysis literature. That is, the confounding of componential definitions that include the union of all non-zero (that is, in traditional terms, all) features on a given dimension with those that contain only the zero feature (in traditional terms, no feature). E.g., in usual notation, an expression such as

$$a_1 b_2 c$$

is ambiguous as between

- (a)  $d_1^{(1)} \cap d_2^{(2)} \cap d_0^{(3)}$
- (b)  $d_1^{(1)} \cap d_2^{(2)} \cap (d_1^{(3)} \cup d_2^{(3)} \cup \dots \cup d_m^{(3)})$ .

The same ambiguity is frequently carried by the expression

$$a_1 b_2$$

where the dimension  $C (=D^{(3)})$  is previously given as part of  $SS$ .

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Note 1. Most of our primitives are borrowed from Carnap (1956). He describes predicates as "predicate expressions in a wide sense, including class expressions." (pp. 6-7) "By the intension of the predicator 'P' we mean the property [attribute] P; by its extension we mean the corresponding class [set]." (p. 18)

Note 2. A partition of a set  $S$  is a division of  $S$  into subsets such that every member of  $S$  is included in exactly one of the subsets. The subsets so created are often called the "cells" of the partition. The Senate of the United States as presently constituted may be partitioned into fifty cells, corresponding to the fifty states; each cell contains two members. The House of Representatives may be partitioned by the same criterion into fifty cells, but the cells contain varying numbers of members.

Note 3. By logically false we mean "L-false" in the meaning given by Carnap (1956:11, 2-3.a.). The general idea of a sentence being L-false is that it "cannot possibly be true" (p. 11); it is false by virtue of the language it is expressed in ( $S_1$ ), independent of all facts regarding its content (see Carnap 1956:11).

Note 4. The last statement follows from (4) and (5).

Note 5. Definition (9) can be written in extended form, by substituting for each term  $p_j$  its own definition as an intersection of features (see definition 7 and equation 8).

$$(10) \quad C = \bigcup_{j=1}^m \bigcap_{i=1}^n d_{(j)}^{(i)} = \bigcup_{j=1}^m d_{(j)}^{(1)} \wedge d_{(j)}^{(2)} \wedge \dots \wedge d_{(j)}^{(n)} =$$

$$(d_{(1)}^{(1)} \wedge d_{(1)}^{(2)} \wedge \dots \wedge d_{(1)}^{(n)}) \cup (d_{(2)}^{(1)} \wedge d_{(2)}^{(2)} \wedge \dots \wedge d_{(2)}^{(n)}) \cup \dots$$

$$\cup (d_{(m)}^{(1)} \wedge d_{(m)}^{(2)} \wedge \dots \wedge d_{(m)}^{(n)}).$$

The only reason to rewrite definition (9) in expanded form (10) is to emphasize the fact that a semantic category is a union of intersections of semantic features.